

Doppler's Effect (Relativistic)

Let s and s' be the two systems, the latter moving with velocity v along (+)ve direction of x -axis. Let the transmitter and receiver be situated in frames s and s' at $x=0$ and $x'=0$ respectively. Now consider two signals transmitted by the transmitter at time $t=0$ and $t=t_1$, such that there is an interval t_1 between the signals in system s by the receiver situated at $x'=0$. Now according to Lorentz transformation equations we have

$$x' = \frac{x-vt}{\sqrt{1-\beta^2}}$$

$$y' = y$$

$$z' = z$$

$$\text{and } t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\beta^2}}$$

But for the initial signal, i.e. for the first signal $x=0, t=0$

$$\therefore x' = 0 \text{ and } t' = 0$$

This means that in second system the first signal sent at $x=0, t=0$ from system s is received by receiver at s' at $x'=0$, and $t'=0$.

Now for the second signal $x = 0$, $t = t_1$,
the corresponding position in s' is obtained from
first equation of Lorentz transformation, that

$$x' = \frac{-vt_1}{\sqrt{(1-\beta^2)}} \quad \dots \dots \dots \quad (1)$$

and the corresponding time in s' is

Since the receiver is at $x = 0$, therefore the second signal will be received by the receiver when the signal reaches at $x' = 0$ i.e. after travelling a distance [from eqn (1)]

If c is the velocity of light, the time taken by the signal to reach in s' at $x' = 0$,
 (time = $\frac{\text{distance}}{\text{velocity}}$)

$$\delta t' = \frac{vt_1}{c\sqrt{1-\beta^2}}$$

$$= \frac{\frac{v}{c}t_1}{\sqrt{1-\beta^2}} \quad \dots \dots \dots \quad (3)$$

Thus the total time for the reception of two

signal at $x' = 0$ in system $S' = 0$

$$t_1' = t' + \delta t' = [(2) + (3)]$$

$$= \frac{t_1}{\sqrt{(1-\beta^2)}} + \frac{\frac{v}{c} t_1}{\sqrt{(1-\beta^2)}}$$

$$= \frac{t_1 + \frac{v}{c} t_1}{\sqrt{(1-\beta^2)}}$$

$$= \frac{(1 + \frac{v}{c}) t_1}{\sqrt{(1-\beta^2)}}$$

$$= \frac{(1+\beta) t_1}{\sqrt{(1-\beta^2)}}$$

$$\therefore t_1' = t_1 \sqrt{\frac{(1+\beta)}{(1-\beta)}} \quad \dots \dots \dots \quad (1)$$

If we choose t_1 , i.e. time between two signals to be the period of light wave in system S , then t_1' will be the time period to receive the wave in system S' . Thus if ν and ν' are the frequencies of corresponding light waves in system S and S' , we have

$$t_1' = \frac{1}{\nu'} \quad \text{and} \quad t_1 = \frac{1}{\nu} \quad \text{from eqn (2)}$$

$$\frac{1}{\nu'} = \frac{1}{\nu} \sqrt{\frac{(1+\beta)}{(1-\beta)}}$$

$$\nu' = \nu \sqrt{\left(\frac{1-\beta}{1+\beta}\right)}$$

$$\therefore v' = v \sqrt{\frac{\left(1 + \frac{v}{c}\right)}{\left(1 - \frac{v}{c}\right)}} \quad \dots \dots \dots (5)$$

Hence if v is positive i.e. if the receiver is going on away from the transmitter (as considered), from (5) $\omega' < \omega$, this means that the frequency transmitted in system S is greater than that received in system S' , but v is negative if the receiver is coming towards the transmitter $\omega' > \omega$, then the frequency received by the observer is greater than the actual frequency transmitted.

This formula (5) represents the relativistic Doppler effect for light waves in vacuum.

This is Longitudinal Doppler's Effect, Since the observations are made along the direction of travel of light source.

If the observations are made at right angles to the direction of travel of light source, The Doppler effect observed is known as Transverse Doppler effect.

This is found usually in the case of atoms. Unlike the classical theory, the theory of relativity predicts transverse Doppler effect.

For transverse Doppler Effect we have $\delta t' = 0$.
 Since in Lorentz transformations $y = y'$ and $z = z'$.

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